

## The Priestley-Taylor Evaporation Model Applied to a Large, Shallow Lake in the Netherlands

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(Manuscript received 25 September 1978, in final form 27 February 1979)

### ABSTRACT

The applicability of the model of Priestley and Taylor (1972) for evaporation of saturated surfaces is examined for the former Lake Flevo (The Netherlands). This lake had an area of about 460 km<sup>2</sup> and an average depth of 3 m. Daily values of evaporation in the period July–September 1967, determined with the energy-budget method, are compared with the corresponding estimated values obtained by the Priestley-Taylor model. The agreement appears to be satisfactory. The diurnal variation of the parameter  $\alpha$  of the Priestley-Taylor model is found to be pronounced. From standard meteorological observations at Oostvaardersdiep, a station at the perimeter of the lake, and an energy-budget model of Keijman (1974) an indirect extension of the available time series is obtained. In this way energy-budget data for the period April–October 1967 became available. Analysis of this data set leads to the preliminary conclusion that  $\alpha$  has a seasonal variation. This is due to the fact that there is a linear relation between the daily latent heat flux  $LE$  and the equilibrium latent heat flux  $LE_{EQ}$  with a nonzero intercept.

### 1. Introduction

Priestley and Taylor (1972) found that evaporation from saturated surfaces is empirically related to the total energy available for the latent and sensible heat fluxes according to the relation

$$LE = \alpha \frac{s}{s+\gamma} (Q^* - G), \quad (1)$$

where  $E$  is the evaporation,  $L$  the latent heat of vaporization,  $\alpha$  the so-called Priestley-Taylor parameter,  $s$  the slope of the saturation specific humidity-temperature curve,  $Q^*$  the net radiation,  $G$  the surface heat flux,  $\gamma = c_p/L$  and  $c_p$  is the specific heat of air at constant pressure. The term  $s/(s+\gamma)(Q^* - G)$  is often denoted as the equilibrium latent heat flux  $LE_{EQ}$ .

In fact the Priestley-Taylor model implies that  $LE$  is proportional to the first term of Penman's combination equation

$$LE = \frac{s}{s+\gamma} (Q^* - G) + \frac{\gamma}{s+\gamma} f(u) [e_s(T_2) - e_2], \quad (2)$$

where  $f(u)$  is a function of the wind speed  $u$ .

From this it follows that the first and second terms are proportional to each other (Ferguson and Den Hartog, 1975), which implies that  $LE$  is also linearly related to the second term of Penman's equation (De Bruin, 1978).

For saturated land surfaces the soil heat flux  $G$  is small compared to  $Q^*$  when 24 h values are considered. Thus, for this case, the Priestley-Taylor model can be read as (Penman, 1948)

$$LE = \alpha \frac{s}{s+\gamma} Q^*. \quad (3)$$

It is interesting to note that about 20 years ago Makkink (1957) proposed an empirical relation for saturated grass surfaces very similar to (3), viz.,

$$LE = c_1 \frac{s}{s+\gamma} K^{\dagger} + c_2, \quad (4)$$

where  $K^{\dagger}$  is the daily global radiation and  $c_1$  and  $c_2$  are constants ( $c_1$  is approximately 1,  $c_2$  is small, about  $-3 \text{ W m}^{-2}$ ). From a practical point of view (4) is to be preferred to (3), because  $K^{\dagger}$  is much easier to evaluate than  $Q^*$ . On the other hand, it is to be expected that the value of  $c_1$  depends on crop factors such as albedo and roughness.

In advection-free conditions an average value of  $\alpha = 1.26$  was obtained both for water surfaces and saturated land surfaces. Several authors have confirmed this value [Ferguson and Den Hartog (1975); Stewart and Rouse (1976, 1977); Davies and Allen (1973); Mukammal and Neumann (1977)]. Some of these authors found the same value for small, shallow lakes, e.g., Stewart and Rouse (1976, 1977). This is a some-

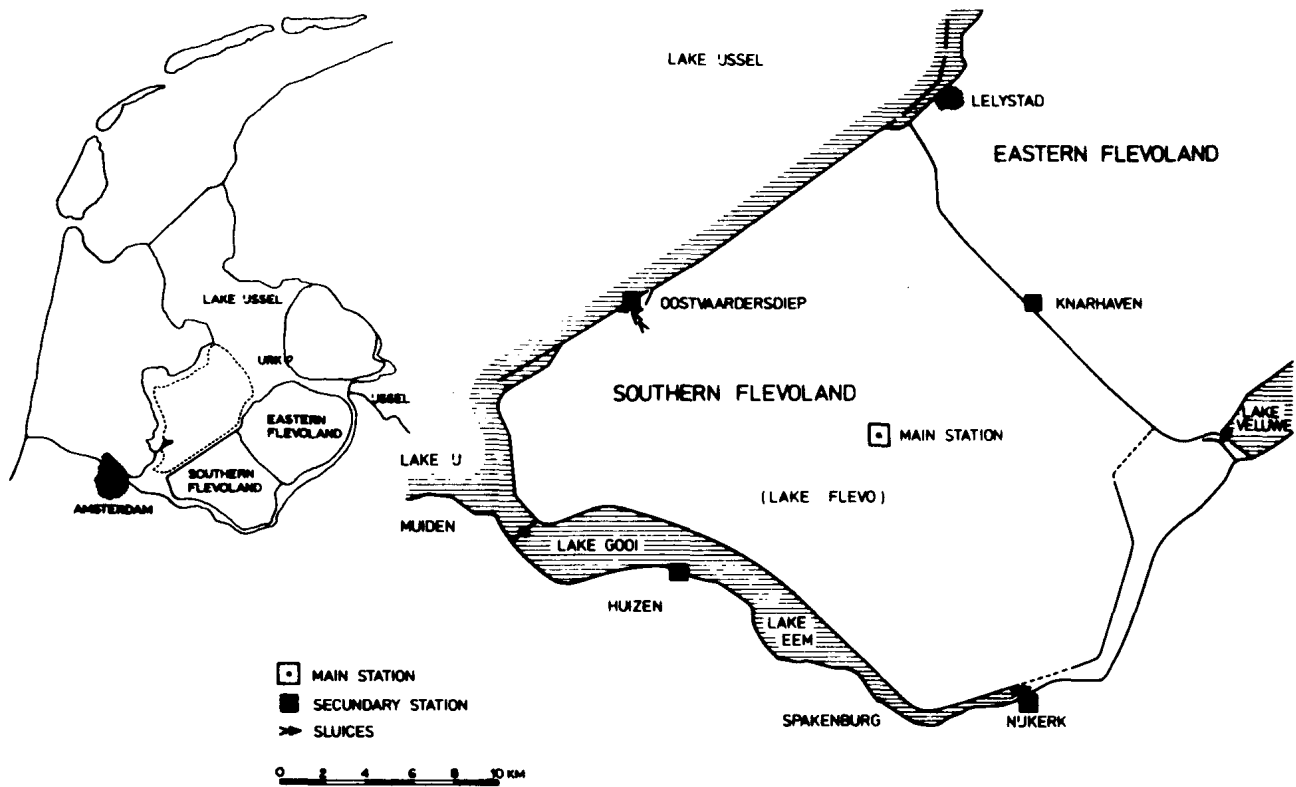


FIG. 1. Map of the Lake Flevo area.

what surprising result, because the influence of advection on the energy budget of small lakes is not negligible. This suggests that there is no need to be unduly strict about the range of applicability of the Priestley-Taylor model. However, this does not hold for very small water surfaces, such as an evaporation pan. In that case advection influences the value of the parameter  $\alpha$  considerably (Mukkammal and Neumann, 1977).

Tsann-Wann Yu (1977) studied the diurnal variation of  $\alpha$  using data of the Wangara experiment (Clarke *et al.*, 1971). His findings, however, apply to an unsaturated surface and will not be considered here.

Combining Eq. (1) with the energy-budget equation

$$LE + H = Q^* - G, \tag{5}$$

(where  $H$  is the sensible heat flux), we obtain a relation between  $\alpha$  and the Bowen ratio  $\beta = H/LE$ :

$$\alpha = \left[ \frac{s}{s + \gamma} (1 + \beta) \right]^{-1}, \tag{6a}$$

or

$$\beta = -\frac{1}{\alpha} \frac{\gamma}{s} + \frac{1 - \alpha}{\alpha}. \tag{6b}$$

Hicks and Hess (1977) analyzed several sets of open water data and found that the empirical relation

$$\beta = 0.63 \frac{\gamma}{s} - 0.15 \tag{7}$$

gives a better fit to the data than Eq. (6b) with  $\alpha = 1.26$ . Eq. (7) leads to

$$LE = \frac{s}{0.85s + 0.63\gamma} (Q^* - G). \tag{8}$$

It is the purpose of this paper to test Eqs. (1), (6a), (6b) and (8). We will use the data collected in the summer and autumn of 1967 during the Flevo project. We will also investigate a possible diurnal and seasonal variation of the parameter  $\alpha$ .

## 2. Experimental

In the framework of a reclamation program of a part of Lake IJssel (The Netherlands) in 1966-67 a dike was built in southern Flevoland. This dike enclosed together with the old coastline a water area of 467.6 km<sup>2</sup>. In this way a temporary lake, called Lake Flevo, was created. Fig. 1 shows a map of the area under consideration. In the summer and autumn of 1967 detailed micrometeorological and hydrological data were collected at this lake.

At the main station, at the center of the lake, net radiation, wind speed at 2, 4 and 8 m, dry and wet bulb temperature at 2 and 4 m, water temperature at several depths, including the water surface, and the heat flux from the water body into the underlying soil were measured continuously. At several places (see Fig. 1) precipitation and water-level changes were

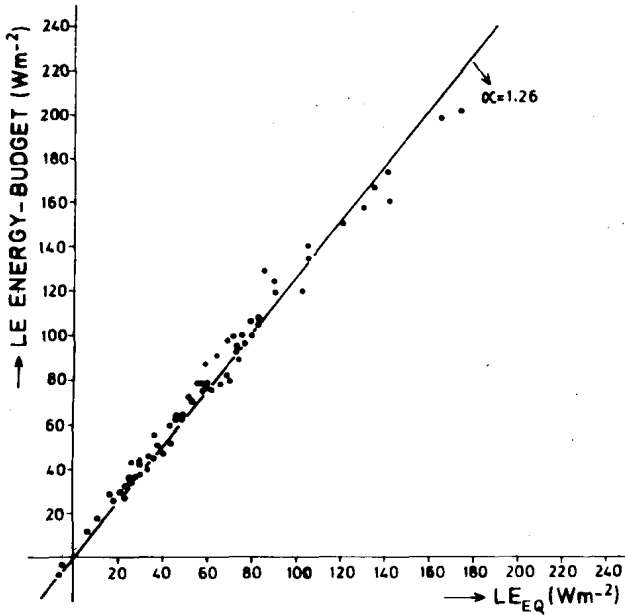


FIG. 2. Test of the Priestley-Taylor model. Comparison of daily average measurements of *LE* and *LE<sub>EQ</sub>*.

determined. At four secondary stations along the perimeter of the lake, standard meteorological data were collected, *viz.*, air temperature and humidity at screen height, wind speed at 3 m and the duration of sunshine. Extensive measurements of turbulent fluctuations of wind and temperature were made (Wieringa, 1973), but these fall outside the scope of this study.

With the data collected at the center of the lake it was possible to determine the evaporation using different methods, while the available data set also allowed calculations of evaporation with the water-budget method. Comparing the evaporation determined with the water-budget method and the evaporation determined with the energy-budget method gives  $0.97 \pm 0.04$  for the average value and the standard error of the ratio of these quantities. This comparison is based on seven water-budget periods with an average length of 4.6 days (Keijman and Koopmans, 1973). Therefore, in this study the energy-budget measurements of *E* are considered the true values of the evaporation. This is a generally accepted procedure (Stewart and Rouse, 1977), which implies that *E* is determined with

$$LE = \frac{Q^* - G}{1 + \beta_{obs}} \tag{9}$$

where  $\beta_{obs}$ , the observed value of the Bowen ratio, is calculated from the usual formula

$$\beta_{obs} = \gamma \frac{T_0 - T_2}{q_s(T_0) - q_2} \tag{10}$$

in which  $T_0$  is the surface temperature,  $T_2$  the air temperature at 2 m,  $q_s(T_0)$  the saturation specific

humidity at  $T_0$ , and  $q_2$  the specific humidity at 2 m. For a given time interval  $\beta_{obs}$  is calculated from the time average value of  $T_0$ ,  $T_2$  and  $q_2$ . The surface skin temperature of a water body, which we will denote by  $T_s$ , is generally lower than the subsurface temperature, denoted above by  $T_0$ . Because the fluxes of sensible and latent heat depend on  $T_s$  rather than on  $T_0$ , one may wonder what the effect would be of using  $T_s$  instead of  $T_0$ . Estimating  $(T_0 - T_s)$  with the model of Hasse (1971), we find a value of approximately  $+0.5^\circ\text{C}$ . This leads to a correction of  $+3\%$  in  $(1 + \beta)^{-1}$ , *LE* and  $\alpha$ . This small correction has been neglected in our calculations.

3. Results

a. Daily values

In Fig. 2 the mean daily values of *LE* measured with the energy-budget method are plotted against *LE<sub>EQ</sub>*. Linear regression calculations forcing the regression line through the origin yield a value of  $\alpha = 1.25 \pm 0.01$ . The corresponding correlation coefficient is 0.991. Defining the error of estimate by the root-mean square (rms) of the differences between measured and calculated values, this error is found to be  $7.0 \text{ W m}^{-2}$ . The regression line with nonzero intercept is  $LE = 1.17 (\pm 0.02) LE_{EQ} + 7.0 (\pm 1.4) [\text{W m}^{-2}]$ . The corresponding error of estimate is  $6.0 \text{ W m}^{-2}$ . This is only slightly smaller than the error of estimate of the zero-intercept line. Therefore, we conclude that the Priestley-Taylor model yields very good results for daily evaporation values at Lake Flevo. As energy-budget measurements of *LE* are only available for July through September, this test holds only for summertime and early autumn.

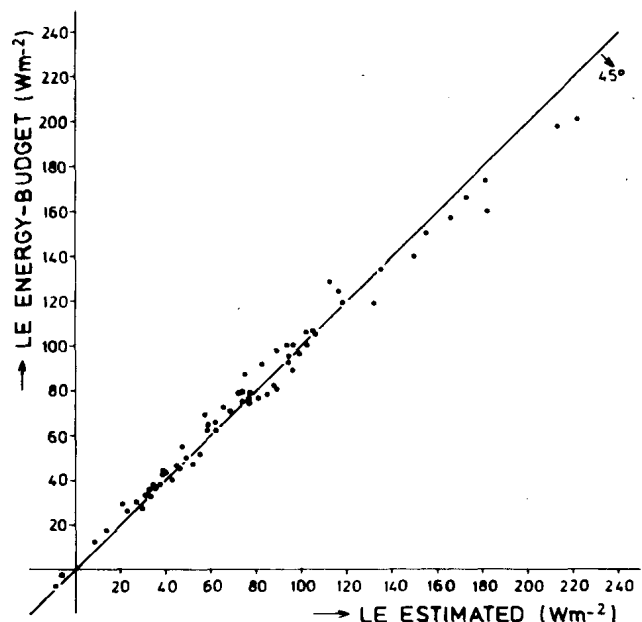


FIG. 3. Test of the model of Hicks and Hess for daily values.

Fig. 3 shows the results of the model of Hicks and Hess (1977), which is a modification of the Priestley-Taylor concept. The slope of the regression line through the origin is 0.98, the correlation coefficient 0.992 and the error of estimate  $6.7 \text{ W m}^{-2}$ . The differences between the two models are small. This is not very surprising. During the test period the mean daily water and air temperature fell in the range  $15\text{--}20^\circ\text{C}$ . In this range the differences between the two models are small.

It should be noted that the observed high correlation between  $LE$  and  $LE_{EQ}$  is partly due to the fact that both quantities contain  $(Q^* - G)$ . This is because we have considered the energy-budget measurements of  $LE$  the true latent heat flux. As mentioned before, this is justified by the fact that these energy-budget measurements compare satisfactorily with the water-balance data. However, the choice of the energy-budget method as a reference implies that scatter due to measuring errors in  $(Q^* - G)$  is not incorporated in the applied

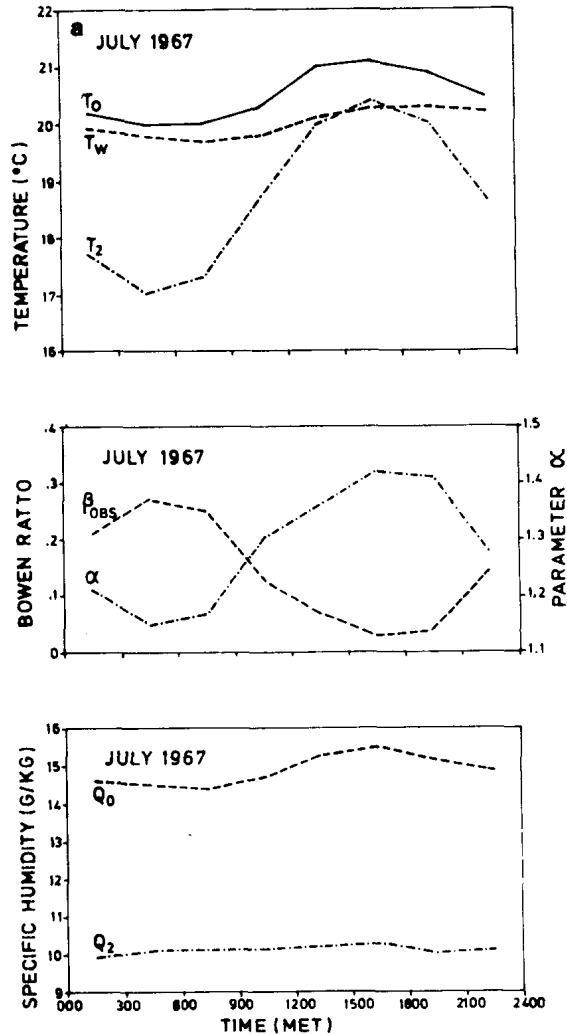


FIG. 4a. Diurnal variation of parameter  $\alpha$ , the Bowen ratio  $\beta_{\text{obs}}$ , surface water temperature  $T_0$ ,  $T_w$  depth-averaged water temperature, air temperature  $T_2$  and specific humidity  $q_2$  over Lake Flevo in July 1967.

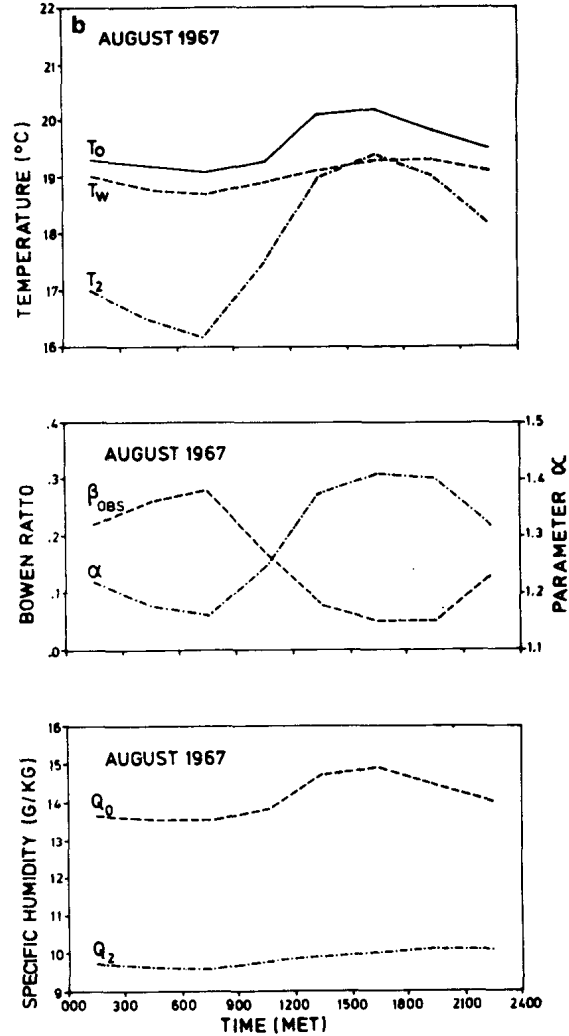


FIG. 4b. As in Fig. 4a except for August 1967.

statistical analysis. Therefore, our results (Figs. 2 and 3) show less scatter than they would have shown if  $LE$  and  $LE_{EQ}$  had had no common factor. This also reflects on the computed standard deviation of  $\alpha$ . However, because the energy-budget method compares well with the water-balance, the rms value of  $\alpha$  itself, as evaluated by our approach, will not be effected significantly by measuring errors in  $(Q^* - G)$ . Therefore, our conclusion that the Priestley-Taylor model yields good results at  $\alpha \approx 1.26$  is not changed by the fact that  $LE$  and  $LE_{EQ}$  both contain  $(Q^* - G)$ .

*b. The diurnal variation of  $\alpha$*

The diurnal variation of  $\alpha$  has been studied with 3 h averages of  $T_0$ ,  $T_2$  and  $q_2$ . We computed  $\beta_{\text{obs}}$  with the aid of Eq. (7) and then  $\alpha$  with Eq. (6). Average values of these quantities for July and August 1967 are shown in Fig. 4. There is a certain day-to-day scatter, which is small, however, so these average values are representative for any given day in the months under consideration. From Fig. 4 it is seen

TABLE 1. The coefficients  $a$  and  $b$  of the regression equation  $LE = aLE_{EQ} + b$ , where both  $LE$  and  $LE_{EQ}$  are calculated per day with the model of Keijman; the corresponding correlation coefficient  $r$  and the least-square estimate of the Priestley-Taylor parameter  $\alpha$  are also shown.

	$a$	$b$ ( $W m^{-2}$ )	$r$	$\alpha$
April	1.32	7.7	0.98	1.50
May	1.07	17.6	0.98	1.28
June	1.15	7.7	0.99	1.25
July	1.08	11.9	0.99	1.21
August	1.09	9.5	0.99	1.20
September	1.18	4.5	0.99	1.25
October	1.24	17.4	0.98	1.49

that in both months the parameter  $\alpha$  has a pronounced diurnal variation with a minimum early in the day and a maximum in the late afternoon. The minima are 1.15 and 1.16 for July and August, respectively, and the maxima 1.42 and 1.41. The daily average value of  $\alpha$  for both months is 1.29. This is in good agreement with the regression calculation of the preceding paragraph, based on mean daily values. As the temperature function  $(s+\gamma)/s$  is nearly constant during the day, the variation of  $\alpha$  is due to the variation of  $\beta_{obs}$ . It can be seen from Fig. 4 that the variation of  $\beta_{obs}$  is, in turn, chiefly due to the variation in  $T_0 - T_2$  because  $q_0 - q_2$  is nearly constant.

The diurnal variation of the air temperature  $T_2$  is much larger than the variation of the surface water temperature  $T_0$ . It is noted that the air temperature never exceeds the surface water temperature. This results in a Bowen ratio which is always positive with a minimum in the late afternoon. This variation of  $\beta_{obs}$  leads to the variation of  $\alpha$  already mentioned above.

### c. The seasonal variation of $\alpha$

The Priestley-Taylor model has only been tested in summertime and early autumn. This holds also for the test given in Section 3. So it is certainly possible that  $\alpha$  varies in the course of the year. Unfortunately, no direct measurements of  $LE$  and  $LE_{EQ}$  are available for the other seasons. In order to get an impression of a possible seasonal variation of  $\alpha$  we followed an indirect method—we used an energy-budget model with standard meteorological observations as input. These data were collected at Oostvaardersdiep in April–October 1967. As is seen from Fig. 1, Oostvaardersdiep was completely surrounded by water and thus it can be considered representative for Lake Flevo. No data were available for a complete year.

From the daily average values of air temperature, air humidity, wind speed and duration of sunshine of this station the terms of the energy-budget equation (5) were calculated with a model developed by Keijman (1974). This model has a solid physical basis, but

contains one important simplification. It is assumed that the water body has no thermal stratification. However, in summertime, when a thermal stratification is most likely to occur, the model yields good results for Lake Flevo (Keijman, 1974). This can be explained by the fact that Lake Flevo has a depth of only a few meters. Therefore, it can easily be stirred by the wind. For further information on the model the reader is referred to Keijman (1974).

For each month we applied linear regression techniques to the daily values of  $LE$  and  $LE_{EQ}$ . This yields the coefficients  $a$  and  $b$  of the regression lines  $LE = aLE_{EQ} + b$ , the correlation coefficients  $r$  and the least-square estimates of  $\alpha$ , the latter obtained by forcing the regression lines through the origin. The values of  $a$ ,  $b$ ,  $\alpha$  and  $r$  are listed in Table 1. Furthermore  $\alpha$  is plotted per month in Fig. 5. It is seen from Table 1 that the correlation coefficients are at least 0.98. Thus, like the Priestley-Taylor model, the energy-budget model predicts a good linear relation between  $LE$  and  $LE_{EQ}$ .

Fig. 5 shows a rather pronounced seasonal variation of  $\alpha$ . In May–September its value differs only slightly from 1.26, but in April and October  $\alpha$  is about 1.50. This is due to the fact that the intercept  $b$  is positive for all months (see Table 1). In summertime the daily values of  $LE$  are large compared to  $b$ , which means that then the deviation of the Priestley-Taylor model will be small. This is no longer the case in spring and autumn, due to the smaller values of  $LE$ . The result is that, if one stays with the Priestley-Taylor model and estimates  $\alpha$  from a regression line forced through the origin, an  $\alpha$  is found with a seasonal variation as given in Fig. 5.

Summarizing, we can say that the energy-balance model used in this study predicts a linear relation between  $LE$  and  $LE_{EQ}$ , just like the Priestley-Taylor

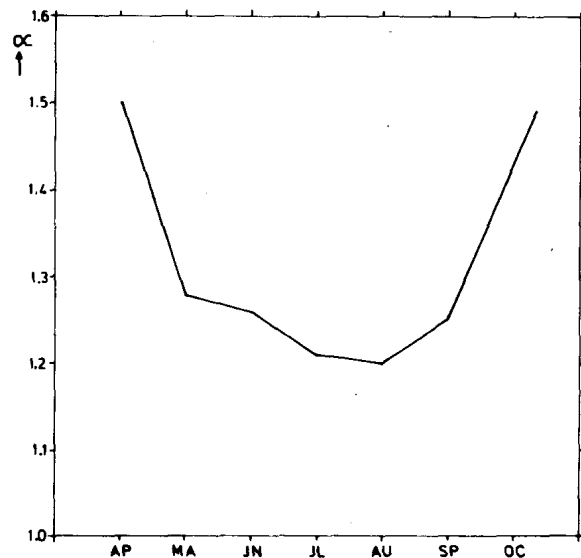


FIG. 5. The seasonal variation of  $\alpha$  as predicted by the model of Keijman.

concept, but with a positive intercept. This positive intercept causes a seasonal variation of the Priestley-Taylor parameter.

#### 4. Discussion

Our results can be summarized as follows:

- 1) In summertime, over 24 h periods the Priestley-Taylor model yields quite satisfactory results at  $\alpha = 1.26$ .
- 2) There is a seasonal variation of  $\alpha$  due to the fact that the regression line between daily values of  $LE$  and  $LE_{EQ}$  has a nonzero intercept.
- 3) The parameter  $\alpha$  shows a diurnal variation.

The first result substantiates the findings of Priestley and Taylor (1972) and Stewart and Rouse (1977). Their results, however, are related to much smaller lakes ( $\approx 0.1$ – $35$  km<sup>2</sup>). We are tempted to conclude, therefore, that in summertime the Priestley-Taylor model, with  $\alpha = 1.26$ , is applicable to all lakes regardless their size. Strictly speaking, this conclusion is contrary to the original concept proposed by Priestley and Taylor because they restricted themselves to advection-free conditions. Small water bodies are strongly influenced by advection, while even large water bodies like Lake Flevo are to a certain extent affected by it. If there exists a general mechanism which at the moment is not understood and which leads to the Priestley-Taylor model at  $\alpha = 1.26$  for advection-free conditions, then from our first and third results we may conclude that over 24 h periods the advective influences are smoothed out (in summer), and that the observed diurnal variation of  $\alpha$  is due to advection. The latter conclusion implies that a diurnal variation of  $\alpha$  will depend on the advective influences and on how the water body reacts on these influences. Thus it is to be expected that the diurnal variation of  $\alpha$  will depend on the size and depth of the lake and on the specific climatological conditions of the region in which the lake is situated.

What are the conditions for finding a diurnal variation of  $\alpha$  of the type observed at Lake Flevo? The diurnal variation of air temperature over the surrounding land must be large compared to the variation of the water temperature and there must be sufficient advection of heat from land to lake. The diurnal variation of the water temperature will be small if the water depth is at least a few meters and there is sufficient mixing to prevent a thermal stratification. The diurnal variation of the air temperature over land depends on many factors of climate, soil and vegetation, but is generally much larger than a few degrees. As these conditions will be met for many lakes, we conclude that the parameter  $\alpha$  will often have a variation similar to that at Lake Flevo.

Our second result implies that a linear relation of the type  $LE = aLE_{EQ} + b$  better fits the data than the original Priestley-Taylor concept. We did not have

enough data to calculate reliable values of the parameters  $a$  and  $b$  per month, but  $a$  is somewhat less than 1.26 and  $b$  is of the order of  $10$  W m<sup>-2</sup>. In summer, when  $LE$  is much larger than  $b$ , the difference between the one-parameter and two-parameter models is not significant.

*Acknowledgments.* This project was a cooperative undertaking from the Board of the Zuidersea Works, Rijkswaterstaat (Mathematical-Physical Branch) and the Royal Netherlands Meteorological Institute (K.N.M.I.). Cooperation and assistance received from the various branches of these Authorities is gratefully acknowledged.

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