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THE EVALUATION OF PENMAN'S NATURAL EVAPORATION FORMULA BY ELECTRONIC COMPUTER*

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Introduction

In experimental agriculture it is often important to be able to estimate the evaporation from an experimental area, and the formula for natural evaporation originally given by Penman (1948) is widely used for this purpose. This formula may be defined by:

$$E_a = 0\cdot35(e_a-e_d)(1+u_2/100) \tag{1}$$

$$H = 0\cdot95R_A(0\cdot18+0\cdot55n/N)-\sigma T_a^4(0\cdot56-0\cdot09\sqrt{e_d})(0\cdot10+0\cdot90n/N) \tag{2}$$

$$E_0 = (0\cdot27E_a+\Delta H)/(0\cdot27+\Delta), \tag{3}$$

where e_a = saturation vapour pressure (mm of mercury) at mean air temperature,
e_d = mean vapour pressure,
u_2 = mean run-of-the-wind at a height of 2 m (miles/day),
R_A = theoretical total daily solar radiation at the surface assuming a completely transparent atmosphere (in mm evaporation equivalent),
n/N = actual/possible duration of bright sunshine,
σT_a^4 = theoretical black-body radiation at mean air temperature, T_a°K,
Δ = slope of vapour-pressure curve at mean air temperature (mmHg/°F),
E_0 = theoretical evaporation for an open water surface (mm/day).

The constants in equations (1) and (2) are empirical and are open to revision; e.g. Penman (1956), Monteith (1961).

The evalutation of E_0 by equation (3) therefore depends only on the standard meteorological data as measured on weather stations. If either a solarimeter to measure the short-wave radiation, which is approximated by $R_A(0\cdot18+0\cdot55n/N)$, or a net radiometer to measure the net radiation approximated by H is available, then the evaluation of E_0 is simplified and more accurate. For the present purpose it is assumed, however, that these additional measurements are not available.

The only elements in equations (1), (2), and (3) which are not directly recorded are e_a, e_d, Δ, R_A, and N, and after evaluation of these quantities the calculation of E_0 by electronic computer is trivial. All these quantities would be found from tables if the calculation were being carried out by hand, but when the calculation is being done by an electronic computer it may be quicker to generate the above five quantities separately for each calculation, since, unless sufficient core storage is available to contain the tables, time must be spent on the relatively slow operation of transferring information from the backing store to the working store. It is shown below that all the above five quantities can be evaluated from simple formulae to an accuracy adequate for any practical application of the results.

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Engelbrecht (1959) has written a program for an IBM 650 computer and this program uses tables which must be read into the computer at the beginning of each run of data.

Young (1963) has developed a program for a Ferranti "Pegasus" computer. He has developed an approximation for e_a (and hence Δ) and calculated R_A and N from δ, the declination of the Sun which is dependent on season only. The approximation given below for e_a is an alternative to that derived by Young (1963). Approximations are also given below for $\sin \delta$ and $1/r^2$, where r is the distance from the centre of the Earth to the centre of the Sun expressed in terms of the length of the semi-major axis of the Earth's orbit. Young (1963) uses a table of monthly values of δ and ignores the variation in r (giving a maximum error of 3% in R_A).

Calculation of Vapour Pressure

The values of saturation vapour pressure were taken from Tables 95 and 97 of Smithsonian Meteorological Tables (1951) where pressure is given in in. of mercury and temperature in degrees Fahrenheit. For temperature in the range 3 to 32°F a cubic polynominal in temperature gave values of saturation vapour pressure over ice which were within 0·0001 in.Hg of the correct value and the relative error never exceeded 0·1% of the correct value. For temperatures in the range 32 to 120°F a quartic polynominal in temperature (fitted by minimizing the relative error) gave values of saturation vapour pressure over water with a greatest relative error of 0·09% which occurred at the extremes of the range. For temperatures within the range 33 to 117°F, the relative error did not exceed 0·05%.

After conversion to mm of mercury the formulae are:

$$3 \text{ to } 32°\text{F}: \quad e_i = 0{\cdot}9479 + 0{\cdot}05352T + 0{\cdot}000{,}812{,}8T^2 + 0{\cdot}000{,}033{,}12T^3 \qquad (4)$$

$$32 \text{ to } 120°\text{F}: \quad e_w = 2{\cdot}1565 - 0{\cdot}034{,}111T + 0{\cdot}003{,}960{,}84T^2 - 0{\cdot}000{,}029{,}091{,}6T^3 + 0{\cdot}000{,}000{,}398{,}647T^4. \qquad (5)$$

By differentiation of equations (4) and (5), Δ is obtained. The vapour pressure e_d may be calculated from the saturation vapour pressure at the wet bulb temperature and the wet bulb depression using a constant dependent on the psychrometer used (Meteorological Office 1940).

After conversion to degrees Centigrade equations (4) and (5) become:

$$-16 \text{ to } 0°\text{C}: \quad e_i = 4{\cdot}5778 + 0{\cdot}37305T + 0{\cdot}012{,}931T^2 + 0{\cdot}000{,}193{,}09T^3 \qquad (6)$$

$$0 \text{ to } 49°\text{C}: \quad e_w = 4{\cdot}5855 + 0{\cdot}32808T + 0{\cdot}011{,}720T^2 + 0{\cdot}000{,}127{,}93T^3 + 0{\cdot}000{,}004{,}184{,}8T^4. \qquad (7)$$

Calculation of R_A and N

R_A is calculated from the formula:

$$R_A = \frac{1440 J_0}{\pi L r^2} [h \sin \phi \sin \delta + \sin h \cos \phi \cos \delta],$$

where r = distance from the centre of the Earth to the centre of the Sun expressed in terms of the length of the semi-major axis of the Earth's orbit,

J_0 = solar constant,

L = latent heat of evaporation,

ϕ = latitude of station,

$h = \cos^{-1}\left[\frac{\sin(-50')}{\cos\phi\cos\delta} - \tan\phi\tan\delta\right]$ corresponding to the Sun's semi-diameter being 16′ and a constant refraction of 34′.

The daylength N is $24h/\pi$ hr.

Thus it is sufficient to find approximations for δ and r. These quantities vary during the year and from year to year, but the yearly variation may be ignored for the present purpose. The values of δ and r were taken from Table 169 of Smithsonian Tables (1951) corresponding to the year 1950. Both these quantities are cyclic with a period of 1 year and an angle θ was therefore defined in terms of d the day number by

$$\theta = 2\pi(d-172)/365,$$

such that θ has a range of 2π over the year and is zero arbitrarily on June 21. Sin δ and $1/r^2$ were Fourier analysed in terms of θ and the following approximations found:

$$\sin\delta = 0{\cdot}00678+0{\cdot}39762\cos\theta+0{\cdot}00613\sin\theta-0{\cdot}00661\cos 2\theta-0{\cdot}00159\sin 2\theta, \quad (8)$$

$$1/r^2 = 1{\cdot}00011-0{\cdot}03258\cos\theta-0{\cdot}00755\sin\theta+0{\cdot}00064\cos 2\theta+0{\cdot}00034\sin 2\theta. \quad (9)$$

The greatest error in $\sin\delta$ was 5×10^{-4}, corresponding to an error in δ of 2′. This gives a maximum error of half a minute of time in the estimated day length at latitude 50°N. A greater error in day length is that due to year-to-year variations over the 4 year cycle giving a maximum error in the estimated day length of 2 min at 50°N. Allowing for both the above sources of error, the greatest relative error is 0·4% at 50°N., and the approximation for $\sin\delta$ is therefore adequate. The effect of longitude may be allowed for by adjusting θ, but if ignored, would introduce an error of not more than 2 min in the estimated daylength at 50°N.

The greatest error in $1/r^2$ was 1×10^{-4}, a relative error of only 0·01%, and hence the relative error in R_A is of the same order as that of day length.

A Computer Program

The author has incorporated the above approximations in a Mercury Autocode program which evaluates Penman's formula for fixed periods of days. The empirical constants of equations (1) and (2) may be set as required and the formula may be evaluated for several sets of constants in one run. If the short-wave radiation is measured directly then this figure may be added to the data and used instead of $R_A(0{\cdot}18+0{\cdot}55\,n/N)$.

A copy of the program and running details are available from the author.

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References

ENGELBRECHT, H. H. (1959).—Application of high speed computers in irrigation research. *Bull. Amer. Met. Soc.* **40**: 566–70.

METEOROLOGICAL OFFICE (1940).—Hygrometric tables for the computation of relative humidity, vapour pressure, and dew point readings of dry and wet bulb thermometers exposed in Stevenson screens. M.O. 265. 4th Ed.

MONTEITH, J. L. (1961).—An empirical method for estimating long-wave radiation exchanges in the British Isles. *Quart. J.R. Met. Soc.* **87**: 171–9.

PENMAN, H. L. (1948).—Natural evaporation from open water, bare soil and grass. *Proc. Roy. Soc.* A **193**: 120–45.

PENMAN, H. L. (1956).—Evaporation. An introductory survey. *Neth. J. Agric. Sci.* **4**: 9–29.

SMITHSONIAN METEOROLOGICAL TABLES (1951).—Sixth revised edition. (Washington.)

YOUNG, C. P. (1963).—A computer programme for the calculation of mean rates of evaporation using Penman's formula. *Met. Mag.* **92**: 84–9.